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BINOMIAL ACCEPTANCE SAMPLING PLANS FOR EVALUATING OIL-WATER SEPARATOR PERFORMANCE

by

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TECHNICAL REPORT NO. 106-13

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I. INTRODUCTION

There are several oil-water separator systems currently being developed for possible future use aboard Navy ships. In order to be accepted, these systems must meet ~~the following~~ certain specified requirements as specified in Test and Evaluation Master Plan (TEMP) No. 067-1 of March 1981:

In Port — 20 ppm of oil or less in effluent water at least 95% of the time. and

At Sea — 100 ppm of oil or less in effluent water at least 99% of the time.

In order to determine whether a system meets the threshold requirements given above, a binomial acceptance sampling plan is often used. In this type of plan, a specified number of effluent samples are collected and analyzed. Each sample is then classified as either a success (oil content does not exceed threshold) or a failure (oil content exceeds threshold). If the number of failures is not greater than some prespecified number, c, the system is accepted. If the number of failures exceeds c the system is rejected.

Section II of this report describes binomial acceptance sampling and defines the terminology which is commonly used in conjunction with these plans. Section III discusses an alternative scheme which relies on an assumption about the distribution of effluent samples. The first appendix consists of tables which should prove useful in the choice of a particular sampling plan while the second appendix contains a listing of the computer program used to produce those tables.

II. BINOMIAL ACCEPTANCE SAMPLING

In general, a binomial acceptance sampling plan is used to demonstrate whether a system meets a specified binomial type threshold at least 100p% of the time. The parameter p is the required success rate and a "good" system has a success rate that exceeds p. The following notation is commonly used:

α = Producer's Risk. This is the risk of rejecting a good system. Of course, it depends on the true success rate of the system.

β = Consumer's Risk. This is the risk of accepting a system which has a true success rate less than or equal to p. The quantity $(1 - \beta)$ is the confidence level and represents the probability of rejecting a bad system.

n = Sample Size.

c = The number of failures allowed. If the number of failures exceeds c, the system is rejected.

In order to construct a binomial acceptance sampling plan, the required success rate, p, and the desired confidence level, $1 - \beta$, are specified. Values of n and c can then be determined so that the consumer's risk is β if the success rate of the system being tested is exactly p. This guarantees a consumer's risk less than β (or, equivalently, a confidence level greater than $1 - \beta$) if the system success rate is less than p. There is still a choice between alternative plans. For example, if $p = .95$ and $\beta = .10$, two possible plans are:

1. Take 105 samples with two or less failures ($n = 105$, $c = 2$).
2. Take 306 samples with ten or less failures ($n = 306$, $c = 10$).

Both of these plans control the consumer's risk at 10%. However, they differ in the producer's risk for a good system. Suppose the true success rate of the system is .97. Then the producer's risk for the first plan is 61.3% while for the second plan it is only 31.4%.

The tables in Appendix A give binomial acceptance sampling plans for required success rates of 95% and 99%. The confidence levels used are 80%, 85%, and 90%. Plans are given for various values of c and the entries in the table are $1 - \alpha$ (the probability of accepting a good system) for several possible success rates. These values were obtained by using a computer program which is reproduced in Appendix B. The program may be used to obtain similar quantities for situations other than those considered in this report. These values allow the user to choose a plan which properly balances the desire for a low producer's risk against limitations on time and resources which restrict the number of samples which may be collected.

It should be noted that if the system just meets the specification, the producer's risk is $1 - \beta$, which is generally constrained to be large. If the system is only slightly better than required, the producer's risk will still be relatively large. In order to reduce this risk, it is to the producer's advantage to take as many samples as possible.

III. ALTERNATIVE SAMPLING PLANS

The major advantage of binomial acceptance sampling lies in the fact that it requires no assumptions about the distribution of effluent samples. However, there are corresponding disadvantages inherent in this type of plan. First, a fixed number of samples must be taken. Since the samples must be taken under actual operating conditions aboard ship, this is a serious constraint. The second major disadvantage results from the fact that all successes are treated alike. Given two at-sea samples with oil contents of 10 ppm and 90 ppm, respectively, only the fact that both were under the threshold value is recorded. This results in a high producer's risk even for very good systems.

It is possible to avoid some of the disadvantages of binomial acceptance sampling by using a parametric approach to the problem. This type of approach requires that one first make an assumption about the statistical distribution of oil contents in effluent samples. The sampling plan is then dependent on the properties of that distribution. The validity of the plan will of course depend on the initial distributional assumption. If that assumption is invalid, the consumer's risk may not be controlled at the specified level. (It may be either higher or lower than desired, depending on p, β, and the true distribution.)

Previous experience indicates that the amount of oil in effluent samples follows a lognormal distribution. Acceptance sampling plans based on normal and lognormal distributions are discussed in detail in [2]. Also, an investigation has been made of how invalid distributional assumptions affect the confidence levels of those plans. The results

of that investigation are reported in [1]. (The results have been reported in terms of tolerance limits, which can be shown to be equivalent to acceptance sampling plans.)

IV. REFERENCES

[1] Burns, K. C. and Smith, D. E. (1983). "Effect of Nonnormality on Tolerance Limits as Applied to Ship Power Margins," Desmatics, Inc. Technical Report No. 119-1.

[2] King, T. L. and Smith, D. E. (1979). "Tolerance Limits and Variables Sampling Plans: Some Power Calculations for the Normal and Lognormal Distributions," Desmatics, Inc. Technical Report No. 106-7.

APPENDIX A: TABLES

The tables in this appendix give the binomial acceptance sampling plans most likely to be used in the evaluation of oil-water separators. The required success rates are 95% and 99% while the confidence levels considered are 80%, 85%, and 90%. The tables give the maximum number of failures allowed (c) and the necessary number of samples (n). If more than c failures are obtained out of n effluent samples, the system is rejected.

The tables also give the probability of accepting a system $(1 - \alpha)$ for several alternative system success rates. These values may be used to select a particular sampling plan which gives an acceptable producer's risk while not requiring an inordinate number of effluent samples.

		True Success Rate				
c	n	.95	.96	.97	.98	.99
0	32	.194	.271	.377	.524	.725
1	59	.199	.311	.468	.669	.882
2	85	.196	.334	.529	.758	.946
3	110	.194	.354	.579	.821	.975
4	134	.195	.375	.625	.868	.988
5	157	.198	.398	.668	.903	995
6	180	.200	.417	.703	.929	998
7	204	.196	.428	.729	.946	999
8	226	.200	.448	.760	.960	9
9	249	.198	.460	.783	.970	[illegible].000
10	272	.197	.472	.803	.978	1.000
12	316	.200	.502	.841	.988	1.000
14	361	.198	.523	.870	.993	1.000
16	405	.199	.546	.894	.996	1.000
18	449	.198	.566	.913	.998	1.000
20	493	.198	.585	.929	.999	1.000
25	601	.199	.631	.957	1.000	1.000
30	709	.199	.668	.974	1.000	1.000
35	816	.199	.703	.984	1.000	1.000
40	923	.198	.732	.990	1.000	1.000
45	1029	.199	.759	.994	1.000	1.000
50	1135	.199	.783	.997	1.000	1.000

Table 1: Probability of Accepting a Good System; Required Success Rate = 95%, Consumer's Risk = 20%.

		True Success Rate				
c	n	.95	.96	.97	.98	.99
0	37	.150	.221	.324	.474	.689
1	67	.146	.246	.399	.612	.855
2	94	.145	.270	.462	.709	.931
3	119	.149	.295	.520	.784	.968
4	144	.149	.313	.566	.837	.985
5	169	.147	.327	.604	.875	.993
6	193	.147	.344	.641	.906	.996
7	216	.150	.363	.677	.930	.998
8	240	.148	.376	.704	.946	.999
9	263	.149	.391	.732	.959	1.000
10	286	.150	.405	.757	.969	1.000
12	332	.150	.430	.798	.982	1.000
14	378	.148	.451	.832	.990	1.000
16	423	.149	.474	.860	.994	1.000
18	468	.148	.494	.884	.997	1.000
20	512	.150	.516	.904	.998	1.000
25	623	.149	.560	.940	1.000	1.000
30	732	.150	.602	.963	1.000	1.000
35	841	.149	.638	.977	1.000	1.000
40	949	.150	.671	.986	1.000	1.000
45	1057	.149	.700	.991	1.000	1.000
50	1164	.150	.728	.994	1.000	1.000

Table 2: Probability of Accepting a Good System; Required Success Rate = 95%, Consumer's Risk = 15%.

		True Success Rate				
c	n	.95	.96	.97	.98	.99
0	45	.099	.159	.254	.403	.636
1	77	.097	.182	.324	.543	.820
2	105	.099	.204	.387	.649	.911
3	132	.099	.222	.438	.728	.956
4	158	.100	.239	.485	.789	.978
5	184	.098	.252	.524	.835	.989
6	209	.098	.266	.562	.872	.995
7	234	.098	.278	.596	.900	.997
8	258	.099	.293	.630	.923	.999
9	282	.099	.306	.659	.941	.999
10	306	.099	.318	.686	.954	1.000
12	353	.099	.343	.735	.973	1.000
14	400	.099	.364	.775	.984	1.000
16	446	.100	.386	.810	.990	1.000
18	492	.100	.406	.839	.994	1.000
20	538	.099	.424	.864	.997	1.000
25	651	.099	.469	.911	.999	1.000
30	763	.099	.510	.942	1.000	1.000
35	873	.100	.551	.963	1.000	1.000
40	984	.099	.584	.976	1.000	1.000
45	1093	.099	.617	.985	1.000	1.000
50	1202	.100	.647	.990	1.000	1.000

Table 3: Probability of Accepting a Good System; Required Success Rate = 95%, Consumer's Risk = 10%.

		True Success Rate				
c	n	.990	.992	.994	.996	.998
0	161	.198	.274	.379	.525	.724
1	299	.199	.309	.464	.664	.879
2	427	.200	.336	.528	.755	.945
3	551	.199	.357	.579	.819	.974
4	671	.200	.378	.624	.866	.988
5	790	.199	.395	.662	.900	.994
6	906	.200	.413	.697	.925	.997
7	1022	.200	.428	.727	.944	.999
8	1137	.199	.442	.753	.958	.999
9	1251	.199	.456	.777	.968	1.000
10	1364	.199	.470	.798	.976	1.000

Table 4: Probability of Accepting a Good System; Required Success Rate = 99%, Consumer's Risk = 20%.

		True Success Rate				
c	n	.990	.992	.994	.996	.998
0	189	.150	.219	.321	.469	.685
1	337	.149	.248	.399	.610	.853
2	471	.150	.273	.463	.708	.930
3	600	.150	.293	.515	.779	.966
4	726	.149	.311	.559	.832	.984
5	848	.150	.328	.600	.872	.992
6	969	.150	.344	.636	.903	.996
7	1088	.150	.359	.670	.926	.998
8	1206	.150	.373	.698	.944	.999
9	1323	.150	.387	.726	.957	1.000
10	1439	.150	.400	.749	.967	1.000

Table 5: Probability of Accepting a Good System; Required Success Rate = 99%, Consumer's Risk = 15%.

		True Success Rate				
c	n	.990	.992	.994	.996	.998
0	230	.099	.158	.251	.398	.631
1	388	.100	.183	.324	.540	.818
2	531	.100	.203	.382	.643	.908
3	667	.100	.220	.432	.722	.954
4	798	.100	.236	.478	.783	.977
5	926	.100	.251	.519	.830	.988
6	1051	.100	.265	.557	.868	.994
7	1175	.100	.278	.591	.896	.997
8	1297	.100	.291	.625	.919	.999
9	1418	.100	.303	.653	.937	.999
10	1538	.100	.315	.680	.951	1.000

Table 6: Probability of Accepting a Good System; Required Success Rate = 99%, Consumer's Risk = 10%.

APPENDIX B: COMPUTER PROGRAM

The computer program which is given here was used to produce the tables in Appendix A. In order to use the program, two control cards must be specified. The values are read in free format as follows:

Card #1

CR - Consumer's risk

MINC - Minimum value of c for which a sampling plan is to be considered.

MAXC - Maximum value of c for which a sampling plan is to be considered.

PP - Required success rate

NP - Number of alternative success rates (including PP) for which probabilities of acceptance are to be calculated.

Card #2

(NP - 1) alternative success rates (do not include PP) for which probabilities of acceptance are to be calculated.

For each value of c between MINC and MAXC (inclusive) the program finds the correct sample size for the binomial acceptance sampling plan. Probabilities of acceptance are then calculated for each of the alternative success rates.

The program makes use of two IMSL (The International Mathematical and Statistical Library) subprograms. These are MDNRIS and MDBIN. MDNRIS calculates an inverse standard normal probability. That is, it computes Z so that

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} \exp(-t^2/2)dt,$$

where P is an input probability in the interval (0, 1).

This subroutine is used in conjunction with the normal approximation to the binomial distribution to obtain an initial estimate of n. MDBIN computes binomial probabilities. As used here, the inputs are c, n, and 1 - PP. MDBIN outputs the probability of obtaining c or less failures in n samples. This probability may be written as follows:

$$P = \sum_{i=0}^{c} \binom{n}{i} (1-PP)^{i}(PP)^{n-i},$$

where PP is the specified success rate. The source code for these subroutines has not been included here because of copyright regulations.

PROGRAM LISTING

```
      DIMENSION P(100),POWER(100,100),M(100)
      READ (5,*) CR,MINC,MAXC,PP,NP
      READ (5,*) (P(I),I=2,NP)
      P(1)=PP
C
C OBTAIN INVERSE NORMAL PROBABILITY FOR APPROXIMATION.
C
      CALL MDNRIS (PP,Z,IER)
C
C START LOOP OVER NUMBER OF FAILURES.
C
      DO 50 NC=MINC,MAXC
C
C USE NORMAL APPROXIMATION TO GET INITIAL ESTIMATE OF REQUIRED
C SAMPLE SIZE. THEN COMPUTE EXACT BINOMIAL PROBABILITY.
C
         A1=(1-PP)**2
         A2=-PP*(1-PP)*Z*Z-2*(1-PP)*NC
         A3=NC*NC
         R=(-A2+SQRT(A2*A2-4*A1*A3))/(2*A1)
         N=INT(R)
         CALL MDBIN (NC,N,(1-PP),BP,BPE,IER)
C
C SEARCH FOR SMALLEST SAMPLE SIZE SUCH THAT CONSUMER'S RISK IS
C LESS THAN OR EQUAL TO USER SPECIFIED VALUE.
C
         IF (BP .GT. CR) GO TO 20
   10    N=N-1
         CALL MBDIN (NC,N,(1-PP),BP,BPE,IER)
         IF (BP .LE. CR) GO TO 10
         N=N+1
         GO TO 30
   20    N=N+1
         CALL MBDIN (NC,N,(1-PP),BP,BPE,IER)
         IF (BP .GT. CR) GO TO 20
   30    CONTINUE
         M(NC-MINC+1)=N
C
C CALCULATE PROBABILITY OF ACCEPTANCE FOR ALTERNATIVE SUCCESS
C RATES SPECIFIED BY USER.
C
         DO 40 J=1,NP
            CALL MDBIN (NC,N,1-P(J),PO,POE,IER)
   40       POWER(J,NC-MINC+1)=PO
   50    CONTINUE
      WRITE (6,60) PP,CR
   60 FORMAT ('1','THRESHOLD=',F4.3,5X,'CONSUMERS RISK=',F4.3)
      DO 100 K=MINC,MAXC
         WRITE (6,70) M(K-MINC+1),K
   70    FORMAT ('-',' P ',5X,'SAMPLE SIZE=',I4,5X,'C=',I2)
         DO 90 J=1,NP
            WRITE (6,80) P(J),POWER(J,K-MINC+1)
   80       FORMAT ('0',F4.3,8X,F4.3)
   90       CONTINUE
  100    CONTINUE
      STOP
      END
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Binomial acceptance sampling is described and alternative sampling schemes based on the lognormal distribution are discussed. Tables are provided to aid in the choice between alternative binomial sampling plans. In addition, a computer program is given which may be used to extend the tables to other situations.

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